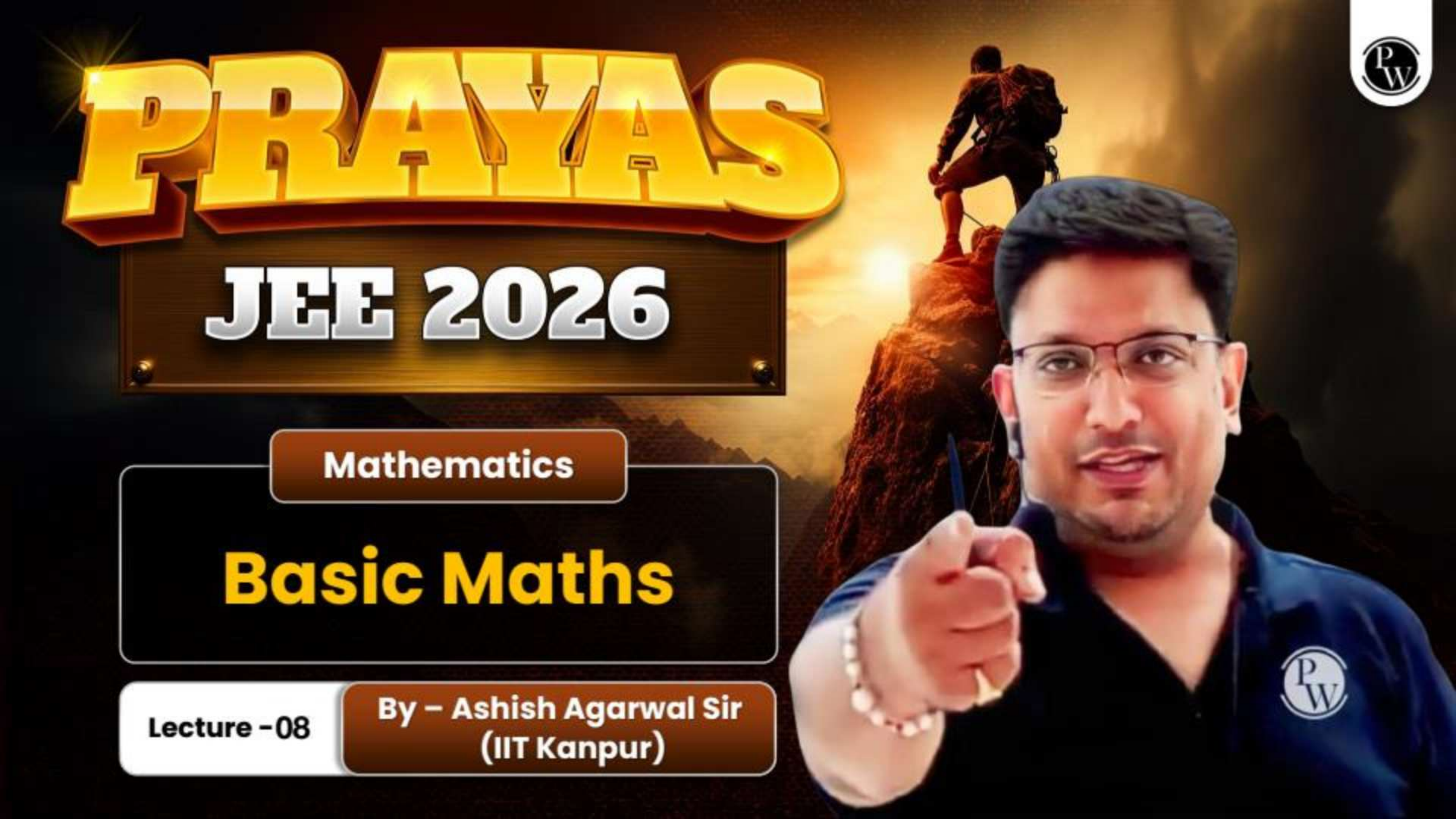
PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture -08
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Topics *to be covered*

A Method of Intervals/ Wavy Curve Method

B Problem Practice

Homework Discussion

QUESTION [IIT-JEE 1980]

(Home Challenge-01)

$$\sqrt{x+y}^{\,3} = \left((x+y)^{\frac{1}{2}}\right)^3 = (x+y)^{3/2}$$

If $x > y > 0$, then show that the expression $\left(\sqrt{2}\left(2x+\sqrt{x^2-y^2}\right)\left(\sqrt{x-\sqrt{x^2-y^2}}\right)\right)$ can be simplified to $\sqrt{(x+y)^3}-\sqrt{(x-y)^3}$.

$$= \left((x+y)^3\right)^{\frac{1}{2}} = \sqrt{(x+y)^3}$$

$$E = \left(2x+\sqrt{x-y}\cdot\sqrt{x+y}\right)\left(\sqrt{2x-2\sqrt{x^2-y^2}}\right)$$

$$= \left(2x+\sqrt{x-y}\cdot\sqrt{x+y}\right)\sqrt{\sqrt{x+y}^2+\sqrt{(x-y)}^2-2\sqrt{x-y}\sqrt{x+y}}$$

$$= \left(x+y+x-y+\sqrt{x-y}\sqrt{x+y}\right)\left(\sqrt{\left(\sqrt{x+y}-\sqrt{x-y}\right)^2}\right)$$

$$= \left|\sqrt{x+y}-\sqrt{x-y}\right|\left(x+y+x-y+\sqrt{x+y}\cdot\sqrt{x-y}\right)$$

$$= \underbrace{\underset{(a\ -\ b)}{\left(\sqrt{x+y}-\sqrt{x-y}\right)}\cdot\underset{(a^2+b^2+ab)}{\left(\sqrt{x+y}^2+\sqrt{x-y}^2+\sqrt{x+y}\cdot\sqrt{x-y}\right)}}_{a^3-b^3} = \left(\sqrt{x+y}\right)^3-\left(\sqrt{x-y}\right)^3 = \sqrt{(x+y)^3}-\sqrt{(x-y)^3}.$$

QUESTION

(KTK 3)

(ADBST)

Given $3x^2 + x = 1$, then the value of $6x^3 - x^2 - 3x$ is equal to

(A) -1

(B) 0

(C) 1

(D) 2

Ans. D

QUESTION

(KTK 7)

Show that the square of $\dfrac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number.

$$V = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{52-2\cdot 5\cdot 3\sqrt{3}}}{10-\sqrt{76+2\cdot 5\cdot \sqrt{3}}} = \frac{\sqrt{5^2+(3\sqrt{3})^2-2\cdot 5\cdot 3\sqrt{3}}}{10-\sqrt{(5\sqrt{3})^2+1+2\cdot 5\sqrt{3}\cdot 1}}$$

$$= \frac{|5-\overset{-ve}{3\sqrt{3}}|}{10-|1+5\sqrt{3}|}$$

$$= \frac{3\sqrt{3}-5}{9-5\sqrt{3}} = \frac{\cancel{3\sqrt{3}-5}}{\sqrt{3}(\cancel{3\sqrt{3}-5})} = \frac{1}{\sqrt{3}} \rightarrow V^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} \in \mathbb{Q}$$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION

Solve: $\dfrac{x^2-5x+12}{x^2-4x+5}>3$

$D=(-4)^2-4.1.5<0$

$a=1, D<0 \Rightarrow$ Denominator is always +ve

$x^2-5x+12>3x^2-12x+15.$

$2x^2-7x+3<0$

$2x^2-6x-x+3<0$

$2x(x-3)-1(x-3)<0$

$(2x-1)(x-3)<0$

$+ \quad - \quad +$

$1/2 \quad 3$

$x\in(1/2,3)$

Lallu: $\dfrac{x^2-5x+12}{x^2-4x+5}-3>0$

$\dfrac{x^2-5x+12-3x^2+12x-15}{x^2-4x+5}>0$

$\dfrac{-2x^2+7x-3}{x^2-4x+5}>0$

$D<0, a>0$ always +ve

$-2x^2+7x-3>0$

$2x^2-7x+3<0$

$(2x-1)(x-3)<0$

$x\in(1/2,3)$

QUESTION

Solve: $\frac{x+1}{x-1} \geq \frac{x+5}{x+1}$

$$\frac{x+1}{x-1} - \frac{x+5}{x+1} \geq 0$$

$$\frac{\cancel{x^2}+2x+1-(\cancel{x^2}+4x-5)}{(x-1)(x+1)} \geq 0$$

$$\frac{-2x+6}{(x-1)(x+1)} \geq 0$$

$$\frac{-2(x-3)}{(x-1)(x+1)} \geq 0$$

$$\frac{x-3}{(x-1)(x+1)} \leq 0$$

$-$ $+$ $-$ $+$

-1 1 3

$x \in (-\infty, -1) \cup (1, 3]$

QUESTION

Solve: $x^2 - 4x + 3 \ge 0$ and $x^2 - 4 \le 0$

Chmatkaari Baba Na Bane

$x^2 \le 4$

$x \le \pm 2$

$(x-1)(x-3) \ge 0$ & $(x-2)(x+2) \le 0$

$x \in (-\infty, 1] \cup [3, \infty)$ & $x \in [-2, 2]$

$\cap$

$x \in [-2, 1]$

set A ke saaray element agar set B mai ho tab A, B kaa subset kehlata hai

$A \subseteq B$

QUESTION

Tah 01

Solve: $(x-1)(x^2+4x+1)(x+2) \le 0$

$D = 16 - 4 = 12 > 0$

$\alpha, \beta = \dfrac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$

$(x-1)(x-(-2+\sqrt{3}))(x-(-2-\sqrt{3}))(x+2) \le 0$

QUESTION

Solve: $(x^2 - x - 6)(x^2 + 6x) \geq 0$ Tah 02

QUESTION

Solve: $x^2 - 5x + 6 \geq 0$ and $x^2 - 10x + 24 \leq 0$ Tah03

QUESTION

$$\frac{x^2 - x - 6}{x^2 + 6x} \geq 0$$

Tahoy

QUESTION

Solve : $(x^2 + 3x + 1)(x^2 + 3x - 3) \geq 5$ Tah 05

let $x^2 + 3x = t$

QUESTION

Find Exhaustive set of values of x satisfying :

(i) $x^3 - 3x^2 - x + 3 > 0$

(ii) $x^4 - 3x^3 - x + 3 < 0$

(iii) $x^4 + 6x^3 + 6x^2 + 6x + 5 \leq 0$

$x^2(x-3) - 1(x-3) > 0$

$(x^2-1)(x-3) > 0$

$(x-1)(x+1)(x-3) > 0$

$-\quad +\quad -\quad +$

$-1 \quad 1 \quad 3$

$x \in (-1,1) \cup (3,\infty)$

Tahos

$(3,\infty)$ ✓

$(-1,1)$ ✓

Ehaustive set of soln
||
complete set of soln.

QUESTION

Solve: $\dfrac{(x+1)(1-2x)(x-1)}{(5-3x)(x-4)} \leq 0$

$$\frac{(x+1)\cancel{(-1)}(2x-1)(x-1)}{\cancel{(-1)}(3x-5)(x-4)} \leq 0$$

$-\quad+\quad-\quad+\quad-\quad+$

$-\infty \quad -1 \quad 1/2 \quad 1 \quad 5/3 \quad 4 \quad \infty$

$$x \in (-\infty,-1] \cup \left[\frac{1}{2},1\right] \cup \left(\frac{5}{3},4\right)$$

why does method of Interval work ?

$P(x) = x-5 \longrightarrow \text{root} = 5$

-ve　　+ve

5

--- root ++

$\text{Ex: } \overset{F_1}{(x-5)}\overset{F_2}{(x-4)}$

+　　−　　+

4　　5

	F_1	F_2
5 ke aagay	+	+
4 se 5 ke Bich	−	+
4 se peechay	−	−

★ Every linear with odd power>1 behaves like as if had power 1

Ex: $(x-3)^3$, $x-3$, $(x-3)^{15}$, $(x-3)^{101}$

★ $(\text{linear in } x)^{\text{even}} \geq 0$

Case of Repeated Factors

Treat as if power is 1

Ex: $(x-1)(x-2)^3(x-4)^5 \geq 0$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & 1 & & 2 & & 4 & \end{array}$$

$x \in [1,2] \cup [4, \infty)$

Noble Factor

Ex: $(x-1)(x-2)^2(x-3) < 0$

$(x-1)(x-3) < 0$ $\quad \begin{array}{ccccc} + & & - & & + \\ & 1 & & 3 & \end{array}$ $\quad x \neq 2$

$x \in (1,3) - \{2\}$

F_1		F_2		
$+$	$\cdot$	$-$	$=$	$-$
$+$	$\cdot$	$+$	$=$	$+$

Kaam Ki Baat

B_1 : Every odd integral power of a linear factor is treated as 1.

Kaam Ki Baat

B_2 : In case of even power of any factor, first we assume that it is always positive. So we delete it from the inequality but in the end we make a direct check at that value of x where the deleted factor is zero.

QUESTION

Noble factor

Solve: $(x + 1)(x - 3)(x - 2)^2 \geq 0$

$(x+1)(x-3) \geq 0$ — $x = 2$ is also possible.

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -1 & & 3 & \end{array}$$

$x \in (-\infty, -1] \cup [3, \infty) \cup \{2\}$.

$0 \geq 0 \rightarrow$ True

$a \geq b$ kaa matlab

$a > b$ or $a = b$

Dono mai se koi kum se kum koi Eak sahi ho jayay toh $a \geq b$ ko sahi kehte hai

Ex: $2 \geq 1$ T

Ex. $3 \geq 3$ T

QUESTION

Noble factor

Solve: $(x+1)(x-3)(x-2)^2 \geq 0$

$\cup$ = or

$\cap$ = and

$0 \geq 0 \to$ True

$a \geq b$ kaa matlab

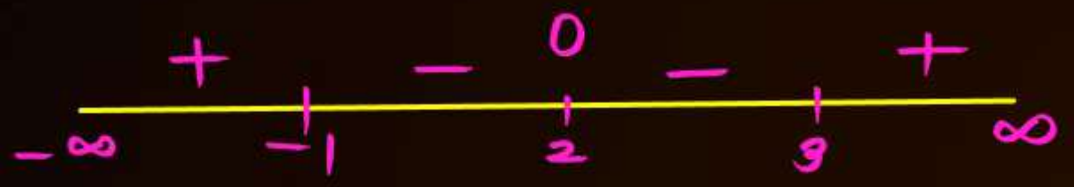
$a > b$ or $a = b$

Dono mai se koi kum se kum koi Eak sahi ho jayay toh $a \geq b$ ko sahi kehte hai

Ex: $2 \geq 1$ T

Ex: $3 \geq 3$ T

M(2)



$x \in (-\infty, -1] \cup [3, \infty) \cup \{2\}$.

QUESTION

Noble Factors

Solve: $(x+4)^5(x+3)^6(x+2)^7(x-1)^8 \le 0$

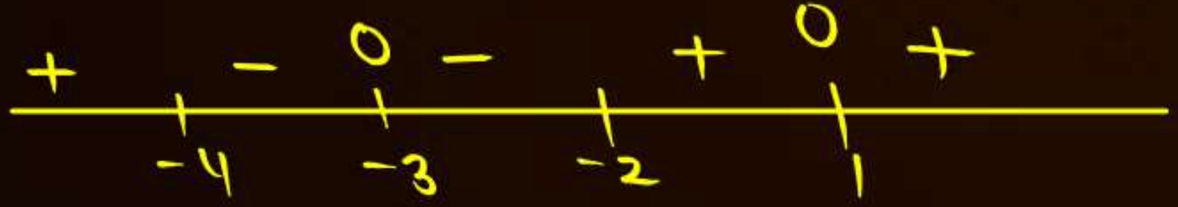
$x+3=0 \qquad x-1=0$

$x=-3 \qquad x=1$

M① $(x+4)^5(x+2)^7 \le 0$, $x=-3, 1$ are also possible

$x \in [-4,-2] \cup \{1\}$

M②



$x \in [-4,-2] \cup \{1\}$

QUESTION

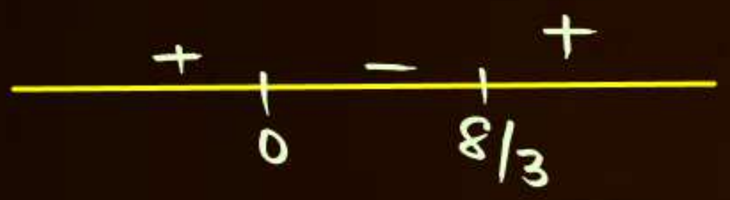
Number of positive integral solution of $\frac{x^3(2x-3)^2(x-4)^6}{(x-3)^2(3x-8)} \leq 0$

$\frac{x^3}{3x-8} \leq 0$, $x = \frac{3}{2}, 4$ are Also possible

Already present

$x = 3$ is N.P

Not in Answer



$x \in [0, 8/3) \cup \{4\}$.

Integer values of $x = 0, 1, 2, 4$

No: of Integral values $= 4 =$ Number of non negative Integral values.

No: of positive Integral Values $= 3$

QUESTION

Solve: $(x-1)^2(x+1)^3(x-4) < 0$ Tah 07

QUESTION

The complete solution set of inequality $\dfrac{(x-5)^{1005}(x+8)^{1008}(x-1)}{x^{1006}(x-2)^3(x-3)^5(x-6)(x+9)^{1010}} \le 0$ is

(A) $(-\infty,-9)\cup(-8,0)\cup(0,1)\cup(2,3)\cup[5,6)$

(B) $(-\infty,-9)\cup(-9,0)\cup(0,1)\cup(2,3)\cup(5,6)$

(C) $(-\infty,-9)\cup(-9,0)\cup(0,1]\cup(2,3)\cup[5,6)$

(D) $(-\infty,0)\cup(0,1]\cup(2,3)\cup[5,6)$

$\dfrac{(x-5)^{1005}(x-1)}{(x-2)^3(x-3)^5(x-6)} \le 0$, $x=-8$ is also possible

$x=0,-9$ are not possible.

$-\;\;1\;\;+\;\;2\;\;-\;\;3\;\;+\;\;5\;\;-\;\;6\;\;+$

$x\in(-\infty,1]\cup(2,3)\cup[5,6)-\{0,-9\}$

OR

$x\in(-\infty,-9)\cup(-9,0)\cup(0,1]\cup(2,3)\cup[5,6)$

$\rightarrow -8$ is here

QUESTION

Solve: $\frac{(x-1)^2(x+1)^3}{x^4(x-2)} \leq 0$

Tah08

QUESTION

Find the exhaustive solutions set of

$$\frac{(2x-5)^{100}(x+3)(2x+1)^{101}}{(x^2-4)^{151}(3x-4)^{197}} < 0$$

Tan09

Ans. $(-\infty,-3)\cup\left(-2,-\frac{1}{2}\right)\cup\left(\frac{4}{3},2\right)$

If a Factor is Eliminated

The facter which is eliminated from Numerator & Den. it's root is never a part of answer

$$\frac{(x-1)(x-2)(x-3)}{(x^2-4)} < 0$$

$$\frac{(x-1)\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x+2)} < 0$$

$$\frac{(x-1)(x-3)}{x+2} < 0 \quad , \quad x \neq 2$$

$x \in (-\infty, -2) \cup (1, 3) - \{2\}$.

OR

$x \in (-\infty, -2) \cup (1, 2) \cup (2, 3)$

QUESTION

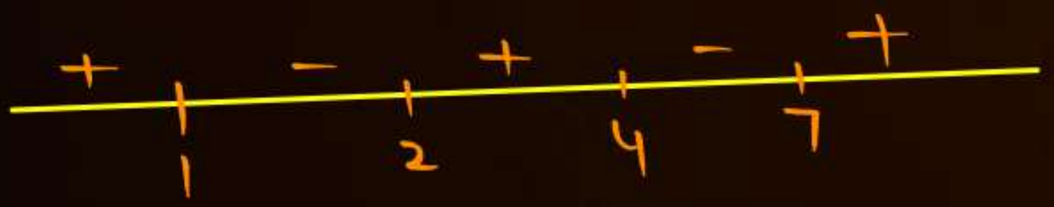
Solve for x : $\dfrac{(x^2-8x+7)(x^2-5x+4)(x-3)^2}{(x^2-3x+2)} \leq 0.$

$$\frac{(x-1)(x-7)\cancel{(x-1)}(x-4)(x-3)^2}{\cancel{(x-1)}(x-2)} \leq 0$$

$$\frac{(x-7)(x-1)(x-4)}{x-2} \leq 0$$

$x=3$ is also possible

$x \neq 1$



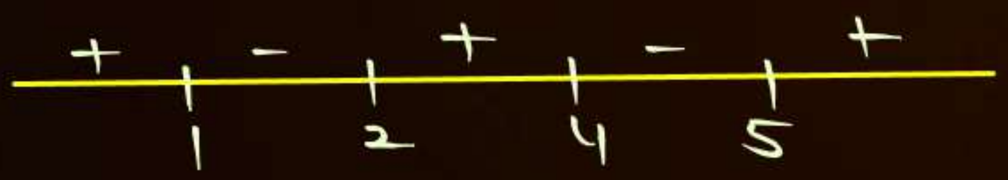
$x \in (1,2) \cup [4,7] \cup \{3\}$

QUESTION

Solve for x: $\frac{(\cancel{x-3})^3(x-1)(x-2)}{(x-4)\cancel{(x-3)}(x-5)} \geq 0$

$$\frac{(x-3)^2\,(x-1)(x-2)}{(x-4)\,(x-5)} \geq 0\ ,\quad x \neq 3$$

$$\frac{(x-1)\,(x-2)}{(x-4)(x-5)} \geq 0\ ,\quad x \neq 3$$



$x \in (-\infty, 1] \cup [2, 4) \cup (5, \infty) - \{3\}$

OR

$x \in (-\infty, 1] \cup [2, 3) \cup (3, 4) \cup (5, \infty)$

QUESTION

Solve: $\dfrac{(x^2-6x+8)^3(x^2-7x+6)^2}{(x^2-5x+6)(x-1)^3} \le 0$

$x^2-7x+6=(x-1)(x-6)$

$$\frac{(x-2)^{\cancel{3}\,2}(x-4)^3\,\cancel{(x-1)^2}(x-6)^2}{\cancel{(x-2)}(x-3)(x-1)^{\cancel{3}}} \le 0$$

$$\frac{\cancel{(x-2)^2}(x-4)^3\,\cancel{(x-6)^2}}{(x-3)(x-1)} \le 0 \qquad x \neq 2, 1$$

$x = 6$ is possible.

$$\frac{(x-4)^3}{(x-3)(x-1)} \le 0$$

$-$ 1 $+$ 3 $-$ 4 $+$

$x \in (-\infty, 1) \cup (3, 4] \cup \{6\}$.

QUESTION

Solve: $\dfrac{(x^2-4x+5)^2(x-3)^2(x+1)^3}{(x-1)(x-5)^3(x^2-7x+12)} > 0$

Tan10

QUESTION

Find the exhaustive solutions set of $\frac{(x-4)(2x-5)^{27}(x^2-9)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0.$ Tan 11

Ans. $(-\infty,-5) \cup (-4,-3) \cup \left(\frac{5}{2},3\right) \cup (\mathbf{3},\mathbf{4}) \cup (\mathbf{5},\infty)$

Double Inequality

QUESTION

Solve the system of in equations $-5 \leq \frac{2-3x}{4} \leq 9$

$$-20 \leq 2-3x \leq 36$$

$$-22 \leq -3x \leq 34$$

$$\frac{-22}{-3} \geq x \geq \frac{34}{-3}$$

$$-\frac{34}{3} \leq x \leq \frac{22}{3}$$

QUESTION

Solve the system of in equations $2x - 1 > x + \frac{7-x}{3} > 2, x \in R$ Jan 12

QUESTION

Solve : $1 < \dfrac{3x^2-7x+8}{x^2+1} \leq 2$

$x^2+1 = x^2+0x+1$

$D = 0^2 - 4\cdot1\cdot1 < 0$

$a = 1 > 0,\ D < 0$

always +ve

$x^2+1 < 3x^2-7x+8 \leq 2x^2+2$

$2x^2-7x+7 > 0$

$D = (-7)^2 - 4\cdot2\cdot7 < 0$

$a = 2 > 0$

$\Downarrow$

$x \in R$

$x^2-7x+6 \leq 0$

$(x-1)(x-6) \leq 0$

$x \in [1,6]$

$\cap$

$x \in [1,6]$

QUESTION

Solve following double inequalities :

Tan13

(i) $-3 < \frac{2x-7}{5} \leq 8$

(ii) $x^2 + 2 \leq 3x < 2x^2 - 5$

(iii) $-2 < \frac{x-5}{2x+1} < 5$

Saari Class Illustrations
Retry karni Hai

Solution to Previous TAH

QUESTION

The square root $5 + 2\sqrt{6}$ is -

(A) $\sqrt{3} + 2$

(B) $\sqrt{3} - \sqrt{2}$

(C) $\sqrt{2} - \sqrt{3}$

(D) $\sqrt{3} + \sqrt{2}$

Q

The Square root of $5+2\sqrt{6}$ is :- Tah-01

$$\sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}\sqrt{2}}$$

$$= \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{3}+\sqrt{2} \ \underline{\text{Ans}}$$

Tah-01.

$$\sqrt{5+2\sqrt{6}} = \sqrt{(1)^2+(\sqrt{6})^2+2\cdot 1\cdot \sqrt{6}}$$

$$= \sqrt{(1+\sqrt{6})^2}$$

$$= |1+\sqrt{6}|$$

$$= 1+\sqrt{6}$$ Ans.

LECTURE - 07

TAH-01

Q) The square root $5+2\sqrt{6}$ is :-

(A) $\sqrt{3}+2$

(B) $\sqrt{3}-\sqrt{2}$

(C) $\sqrt{2}-\sqrt{3}$

(D) $\sqrt{3}+\sqrt{2}$ ✓

Soln $\sqrt{5+2\sqrt{6}}$

$= \sqrt{3+2+2\sqrt{3}\sqrt{2}}$

$= \sqrt{(\sqrt{3}+\sqrt{2})^2} = |\sqrt{3}+\sqrt{2}|$

$= \sqrt{3}+\sqrt{2}$ ans (OPTION-D)

QUESTION [IIT-JEE 1980]

The expression $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

(A) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$

(B) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

(C) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$

(D) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

Tah-02.

$$\frac{12}{3+\sqrt{5}+2\sqrt{2}} = \frac{12}{(3+\sqrt{5})+2\sqrt{2}} \times \frac{(3+\sqrt{5})-2\sqrt{2}}{(3+\sqrt{5})-2\sqrt{2}}$$

$$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{(3+\sqrt{5})^2-(2\sqrt{2})^2} = \frac{12(3+\sqrt{5}-2\sqrt{2})}{14+6\sqrt{5}-8}$$

krish keshri
jharkhand

$$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{6+6\sqrt{5}} = \frac{\overset{2}{\cancel{12}}(3+\sqrt{5}-2\sqrt{2})}{\cancel{6}(1+\sqrt{5})}$$

$$= \frac{6+2\sqrt{5}-4\sqrt{2}}{(1+\sqrt{5})} \times \frac{(1-\sqrt{5})}{(1-\sqrt{5})}$$

$$= \frac{6-6\sqrt{5}+2\sqrt{5}-10-4\sqrt{2}+4\sqrt{10}}{-4}$$

$$= \frac{-4-4\sqrt{5}-4\sqrt{2}+4\sqrt{10}}{-4}$$

$$= \frac{\cancel{-4}(1+\sqrt{5}+\sqrt{2}-\sqrt{10})}{\cancel{-4}}$$

$$= 1+\sqrt{5}+\sqrt{2}-\sqrt{10} \quad \text{Ans.}$$

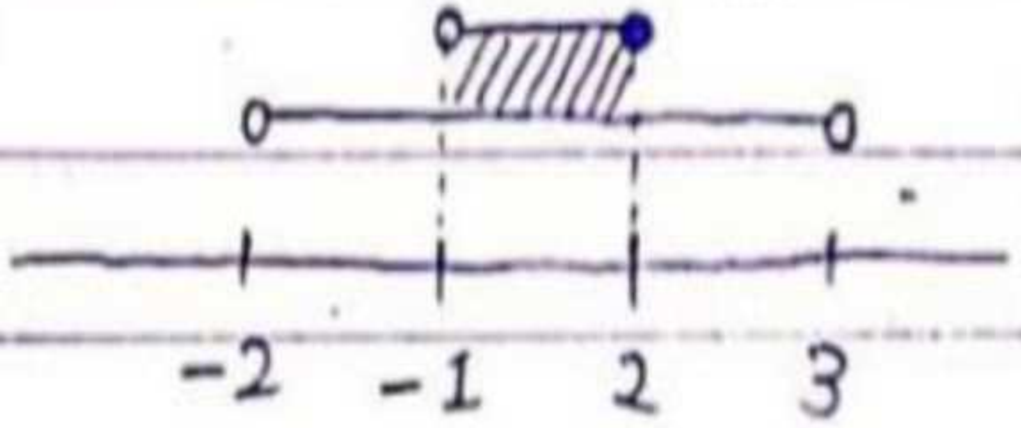
Intersection & Union

Find $A \cup B$ & $A \cap B$

1. $A = (-3, 1]$ & $B = [-4, 0)$
2. $A = (-3, -1)$ & $B = [-1, 2]$
3. $A = (-2, 3)$ & $B = (-1, 2]$

Tah-03.

(A)



$\Rightarrow A \cap B = (-1, 2]$.

$A = (-2, 3)$

$B = (-1, 2]$

$\Rightarrow A \cup B = (-2, 3)$.

QUESTION

Evaluate the following

(i) $(-\infty, 3) \cap [-2, \infty)$

(ii) $(-4, 1] \cap (-3, 4)$

(iii) $(0, 5] \cap (1, \infty)$

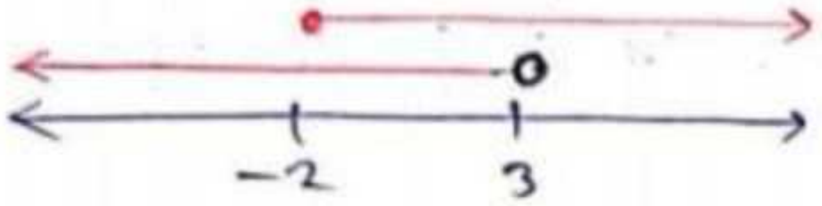
(iv) $[0, 3) \cup [2, 6)$

(v) $[2, \infty) \cup (4, \infty)$

(vi) $[-1, 1) \cup [2, 5]$

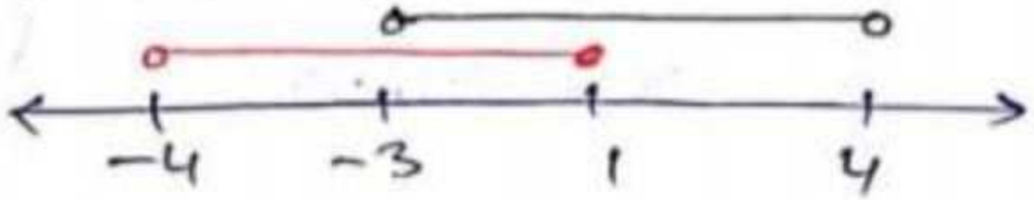
TAH-3B! Evaluate the following!

Sol^n (i) $(-\infty, 3) \cap [-2, \infty)$

$\Rightarrow$ 

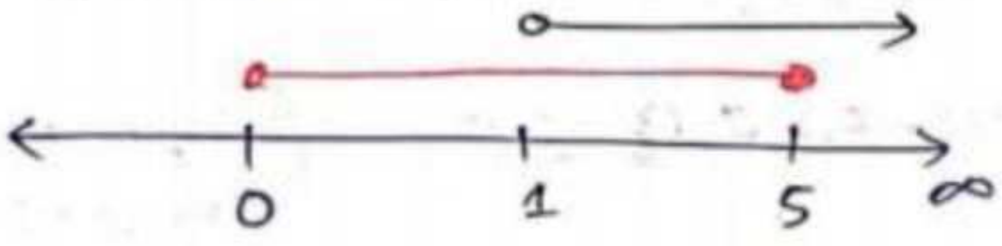
$x \in [-2, 3)$.

(ii) $(-4, 1] \cap (-3, 4)$

$\Rightarrow$ 

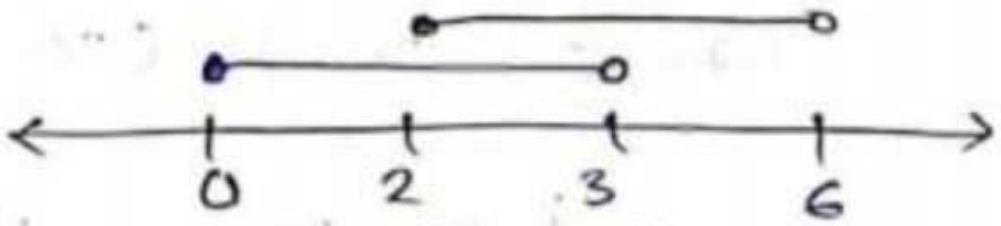
$x \in (-3, 1]$.

(iii) $(0, 5] \cap (1, \infty)$

$\Rightarrow$ 

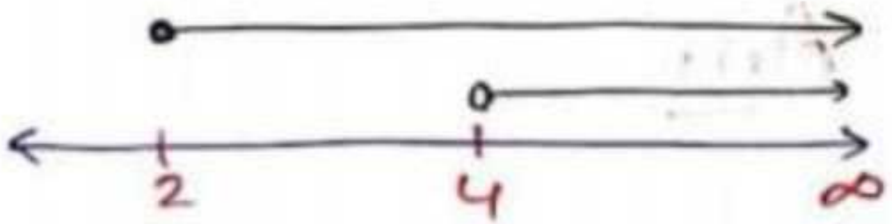
$x \in (1, 5]$.

(iv) $[0, 3) \cup [2, 6)$

$\Rightarrow$ 

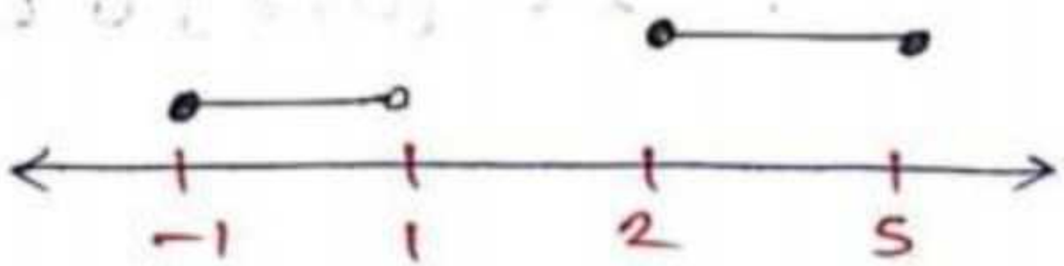
$\therefore x \in [0, 6)$.

(v) $[2, \infty) \cup (4, \infty)$

$\Rightarrow$ 

$x \in [2, \infty)$

(vi) $[-1, 1) \cup [2, 5]$

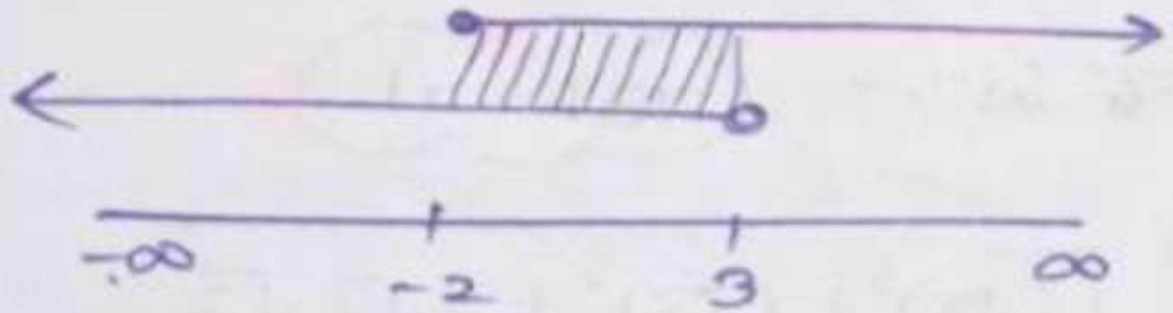
$\Rightarrow$ 

$\therefore x \in [-1, 1) \cup [2, 5]$

Q Evaluate the following :—

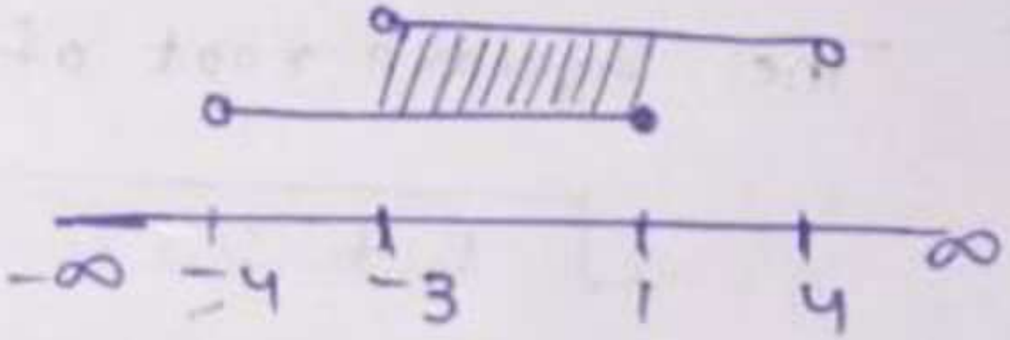
Tah-03 (B)

(i) $(-\infty, 3) \cap [-2, \infty)$



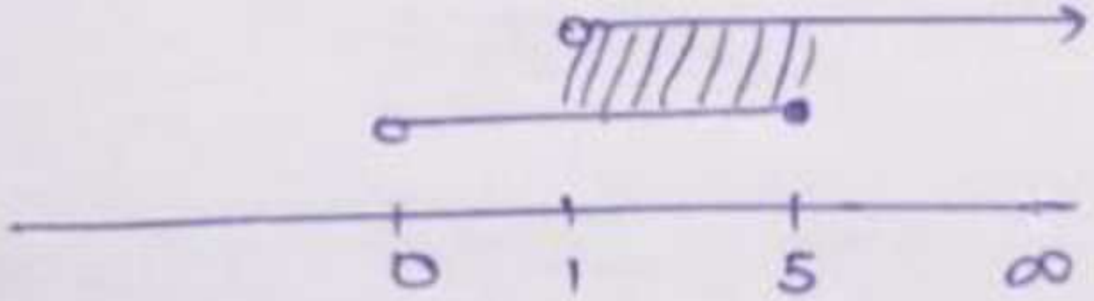
$[-2, 3)$

(ii) $(-4, 1] \cap (-3, 4)$



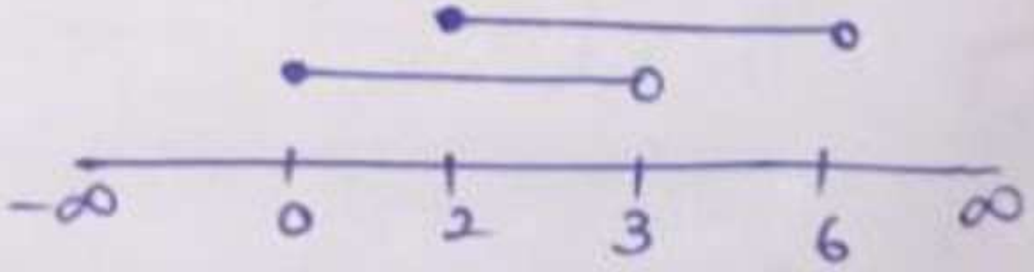
$(-3, 1]$

(iii) $(0, 5] \cap (1, \infty)$



$(1, 5]$

(iv) $[0, 3) \cup [2, 6)$



$= [0, 6)$

QUESTION

$$\frac{2x-3}{5} \geq 1$$

$$\frac{2x-3}{-5} \geq 1$$

$$\frac{x-3}{x-2} \geq 5$$

Q (1) $\frac{2x-3}{5} \geq 1$

$\frac{2x-3}{5} - \frac{1}{1} \geq 0$

$\frac{2x-3-5}{5} \geq 0$

$\frac{2x-8}{5} \geq 0$

$2x-8 \geq 0$

$x-4 \geq 0$

$x \geq 4$

$x \in [4, \infty)$

(2) $\frac{2x-3}{-5} \geq 1$ Tah-04

$2x-3 \leq -5$

$2x \leq -2$

$x \leq -1$

$x \in (-\infty, -1]$

QUESTION

Solve: $x(7 - x)(2 - x)(x - 5) \leq 0$

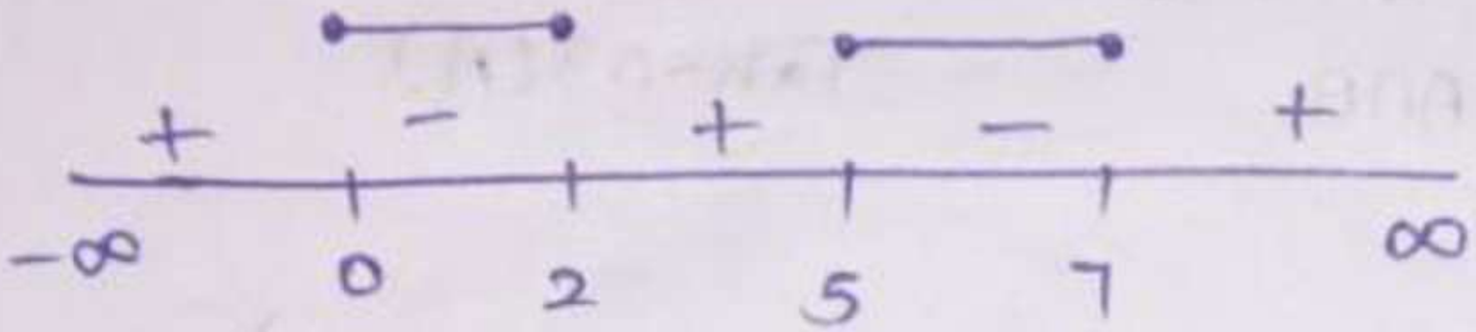
Solve : $x(7-x)(2-x)(x-5) \le 0$

Tah -05

$$x(7-x)(2-x)(x-5) \le 0$$

$$x(-1)(x-7)(-1)(x-2)(x-5) \le 0$$

$$x(x-7)(x-2)(x-5) \le 0$$

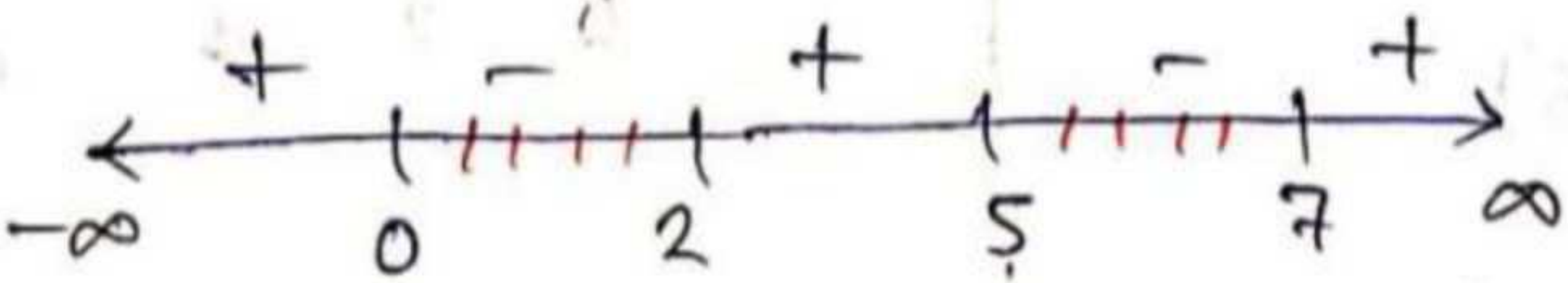


$x \in [0,2] \cup [5,7]$ Ans

TAH-5! $x(7-x)(2-x)(x-5) \leq 0.$

Sol^n $x(7-x)(2-x)(x-5) \leq 0.$

or, $x(x-7)(x-2)(x-5) \leq 0.$ (Sign reverses two times.)



$\therefore x \in [0,2] \cup [5,7].$ Ans.

QUESTION [IIT-JEE 1980]

(Home Challenge-01)

If $x > y > 0$, then show that the expression $\left(\sqrt{2}\left(2x + \sqrt{x^2 - y^2}\right)\left(\sqrt{x - \sqrt{x^2 - y^2}}\right)\right)$ can be simplified to $\sqrt{(x+y)^3} - \sqrt{(x-y)^3}$.

Q-13:- If $x > y \geq 0$, then show that the expression $(\sqrt{2}(2x+\sqrt{x^2-y^2}))(\sqrt{x-\sqrt{x^2-y^2}})$ can be simplified to $\sqrt{(x+y)^3}-\sqrt{(x-y)^3}$.

Soln.- We can write $2x=(x+y)+(x-y)$

now, the expression,

$$E=\{\sqrt{2}(2x+\sqrt{x^2-y^2})(\sqrt{x-\sqrt{x^2-y^2}})\}$$

or, $E=\{(x+y)+(x-y)+\sqrt{x^2-y^2}\}\{\sqrt{2x-2\sqrt{x^2-y^2}}\}$

or, $E=\{(x+y)+(x-y)+\sqrt{x^2-y^2}\}\cdot\{\sqrt{(x+y)+(x-y)-2\sqrt{(x+y)(x-y)}}$

or, $E=\{(x+y)+(x-y)+\sqrt{(x+y)(x-y)}\}\{\sqrt{(\sqrt{x+y})^2+(\sqrt{x-y})^2-2\sqrt{x+y}\cdot\sqrt{x-y}}$

or, $E=\{(x+y)+(x-y)+\sqrt{x+y}\sqrt{x-y}\}\{\sqrt{(\sqrt{x+y}-\sqrt{x-y})^2}\}$

or, $E=\{(x+y)+(x-y)+\sqrt{x+y}\sqrt{x-y}\}\{|\sqrt{x+y}-\sqrt{x-y}|\}$

or, $E=\{(\sqrt{(x+y)})^2+(\sqrt{(x-y)})^2+\sqrt{x+y}\sqrt{x-y}\}\{\sqrt{x+y}-\sqrt{x-y}\}$

or $E=\{(\sqrt{x+y})^2+(\sqrt{x-y})^2+\sqrt{x+y}\sqrt{x-y}\}\{\sqrt{x+y}-\sqrt{x-y}\}$

Let $\sqrt{x+y}=a$ and $\sqrt{x-y}=b$,

then, $E=(a^2+b^2+ab)(a-b)$

or, $E=a^3-b^3$

i.e. $E=(\sqrt{(x+y)})^3-(\sqrt{x-y})^3$

or, $E=\sqrt{(x+y)^3}-\sqrt{(x-y)^3}$

HC 1
by Reed
from West Bengal

Solution to Previous KTKs

QUESTION

(KTK 1)

If a and b are rational numbers and $a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8}$ then find value of $a^2 + b^2 =$

Ans. 274

Q

If a and b are rational numbers and

$a+b\sqrt{2} = 5(\sqrt{2}-3)+\sqrt{8}$, then find value of

$a^2+b^2=$

KTK-01

Sol

$$a+b\sqrt{2} = 5\sqrt{2}-15+2\sqrt{2}$$

$$a+b\sqrt{2} = -15+7\sqrt{2}$$

$$\boxed{a=-15} \quad \boxed{b=7}$$

$$\therefore a^2+b^2 = (-15)^2+(7)^2 = 225+49 = 274$$

Q-5: If a and b are rational numbers and $a+b\sqrt{2} = 5(\sqrt{2}-3)+\sqrt{8}$ then find the value of a^2+b^2.

Soln:

$$a+b\sqrt{2} = 5(\sqrt{2}-3)+\sqrt{8}.$$

or, $a+b\sqrt{2} = 5\sqrt{2}-15+2\sqrt{2}$

or, $a+b\sqrt{2} = -15+7\sqrt{2}$

On comparing,

$$a=-15 \quad \& \quad b=7.$$

So, $a^2+b^2 = (-15)^2+(7)^2$

$= 225+49$

$= 274$. (Ans.)

KTK 1
by Reed
from WB

KTK - 1

$a+b\sqrt{2} = 5(\sqrt{2}-3) + \sqrt{8}$ then find $a^2+b^2=?$

$a+b\sqrt{2} = 5\sqrt{2} - 15 + 2\sqrt{2}$

$a+b\sqrt{2} = -15 + 7\sqrt{2}$

Both side compair

$a = -15, \quad b = 7$

$\therefore \quad a^2+b^2 = (-15)^2 + (7)^2$

$= 225 + 49$

$= 274$

Prince Gupta
Gopalganj (Bihar)

QUESTION

(KTK 2)

If value of $\left(x + \frac{1}{x} = 5\right)$ then find value of :

(i) $x^2 + \frac{1}{x^2}$

(ii) $x - \frac{1}{x}$

(iii) $x^4 + \frac{1}{x^4}$

(iv) $x^4 - \frac{1}{x^4}$

(v) $x^3 + \frac{1}{x^3}$

Ans. (i) 23, (ii) $\pm\sqrt{21}$, (iii) 527, (iv) $\pm 115\sqrt{21}$, (v) 110

Q If value of $\left(x+\frac{1}{x}=5\right)$ then find value of KTK-02

(i) $x^2+\frac{1}{x^2}$

(ii) $x-\frac{1}{x}$

Sol

$x^2+\frac{1}{x^2}+2=25$

$x^2+\frac{1}{x^2}=23$

Sol

$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2=23-2$

$\left(x-\frac{1}{x}\right)^2=21$

$\left(x-\frac{1}{x}\right)=\pm\sqrt{21}$

(iii) $x^4 + \frac{1}{x^4}$

Sol

$$x^2 + \frac{1}{x^2} = 23$$

S.B.S

$$x^4 + \frac{1}{x^4} + 2 = (23)^2$$

$$x^4 + \frac{1}{x^4} = 529 - 2 = 527$$

(v) $x^3 + \frac{1}{x^3}$

Sol

$$x + \frac{1}{x} = 5$$

C.B.S

$$x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right) = 125$$

$$x^3 + \frac{1}{x^3} + 3\,(5) = 125$$

$$x^3 + \frac{1}{x^3} = 125 - 15$$

$$\boxed{x^3 + \frac{1}{x^3} = 110}$$

(iv) $x^4 - \frac{1}{x^4}$

Sol

$$(x^2)^2 - \left(\frac{1}{x^2}\right)^2$$

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

$$= \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= (23)\,(5)\,\pm\sqrt{21}$$

$$= \pm\sqrt{21}\;115 = \pm 115\sqrt{21}$$ Ans

QUESTION

(KTK 3)

Given $3x^2 + x = 1$, then the value of $6x^3 - x^2 - 3x$ is equal to

(A) -1

(B) 0

(C) 1

(D) 2

Ans. D

= ans

Question | KTK - 03

Given $3x^2+x=1$, the the value of $6x^3-x^2-3x$ is equal to

(A)

(B)

(C)

(D)

soln

$$3x^2+x=1$$

$$3x^2=1-x$$

$$6x^3=2x-2x^2$$

$$6x^3-x^2-3x$$

$$=2x-2x^2-3x-x^2$$

$$=-x-3x^2$$

$$=-(x+3x^2)$$

$$=-1$$

= ans

KTK-3! Given $3x^2+x=1$, then the value of
Q-7! $6x^3-x^2-3x$ is equal to:

(a) -1 (b) 0 (c) 1 (d) 2.

Solⁿ: method-1!

$$3x^2+x=1 \text{ ——— (ii)}$$
$$\Rightarrow 3x^2=1-x \text{ ——— (i)}$$

$$6x^3-x^2-3x$$
$$= x[2\cdot 3x^2-x-3]$$
$$= x[2(1-x)-x-3] \quad \text{from (i)}$$
$$= x[2-2x-x-3]$$
$$= x[-3x-1]$$
$$= -3x^2-x$$
$$= -(3x^2+x) \quad \text{from (ii)}$$
$$= -1. \text{ (Ans.)} \qquad \therefore \text{Ans.} \Rightarrow (a)\ -1.$$

method-2!

$$3x^2+x=1$$
$$\Rightarrow 3x^2=1-x \text{ ——— (i)}$$

$$6x^3-x^2-3x$$
$$= 2x\cdot 3x^2-x^2-3x$$
$$= 2x(1-x)-x^2-3x$$
$$= 2x-2x^2-x^2-3x$$
$$= -3x^2-x$$
$$= -(3x^2+x)$$
$$= -1. \text{ (Ans)}$$

KTK-3
By Reed
from W.B.

QUESTION

(KTK 4)

If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) $\frac{1}{3}$

(D) $\frac{5}{3}$

Ans. B

$\boxed{\text{KTK-04}}$

Ques) If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$

(A) $\frac{2}{3}$ (B) $4/3$ (C) $1/3$ (D) $5/3$

$\boxed{\text{sol}^n}$

$$\sqrt{9^x} = \sqrt[3]{9^2}$$

$$9^{x/2} = 9^{2/3}$$

$$\frac{x}{2} = \frac{2}{3}.$$

$$x = \frac{4}{3} \text{ // ans}$$ $\boxed{\text{OPTION-B}}$

Q-8! If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x = ?$

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{5}{3}$.

Sol^n!

$$\sqrt{9^x} = \sqrt[3]{9^2}$$

or $9^{\frac{x}{2}} = 9^{\frac{2}{3}}$

or, $\frac{x}{2} = \frac{2}{3}$

or, $x = \frac{4}{3}$. (Ans.)

QUESTION

(KTK 5)

$$\frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}} =$$

A $x^{76/15}$

B $x^{78/15}$

C $x^{79/15}$

D $x^{77/15}$

Ans. D

Q $\dfrac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}}$

KTK-05

Sol $= \dfrac{(x^3)^{1/2} \times (x^5)^{1/3} \times (x^{77})^{1/30}}{(x^3)^{1/5}}$

$= \dfrac{x^{3/2} \times x^{5/3} \times x^{77/30}}{x^{3/5}} = \dfrac{x^{\frac{3}{2}+\frac{5}{3}+\frac{77}{30}}}{x^{3/5}}$

$= \dfrac{x^{\frac{40+50+77}{30}}}{x^{3/5}} = \dfrac{x^{86/15}}{x^{3/5}} = x^{\left(\frac{86}{15}-\frac{3}{5}\right)} = x^{\frac{77}{15}}$ Ans

Q-9: $\dfrac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}} = ?$

(a) $x^{76/15}$ (b) $x^{78/15}$ (c) $x^{79/15}$ (d) $x^{77/15}$

Soln:- $\dfrac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}}$

$= \dfrac{x^{\frac{3}{2}} \times x^{\frac{5}{3}}}{x^{\frac{3}{5}}} \times x^{\frac{77}{30}}$

$= x^{\frac{3}{2}+\frac{5}{3}-\frac{3}{5}} \times x^{\frac{77}{30}}$

$= x^{\frac{45+50-18}{30}} \times x^{\frac{77}{30}}$

$= x^{\frac{77}{30}} \times x^{\frac{77}{30}}$

$= \left(x^{\frac{77}{30}}\right)^2 = x^{2 \times \frac{77}{30}} = x^{\frac{77}{15}}$

$\therefore$ Ans $\Rightarrow$ (d) $x^{\frac{77}{15}}$

KTK 4,5
by Reed
from WB

QUESTION

(KTK 6)

If $\frac{9^n \times 3^2 \times \left(3^{-n/2}\right)^{-2} - (27)^n}{3^{3m} \times 2^m} = \frac{1}{27}$, where m and n are natural numbers, then find the value of $(m - n)$ is ________

Ans. 1

Page No.

Date /

KTK - 06

If $\frac{9^n \times 3^2 \times 3^{-\frac{n}{2} \times -2} - 27^n}{3^{3m} \times 2^m} = \frac{1}{27}$ where m and n are natural no's, find (m-n).

Sol^n: $\frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^m} \Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^m} \rightarrow \frac{3^{3n}(3^2-1)}{3^{3m} \times 2^m} = \frac{1}{27}$

$\frac{3^{3n} \cdot 2^3 \cdot 3^3}{3^{3m} \cdot 2^m} = 1 \Rightarrow 3^{3n+3} \cdot 2^3 = 3^{3m} \cdot 2^m$

$\Rightarrow 3n+3 = 3m \quad , \; m = 3$

$3n+3 = 9 \Rightarrow 3n = 6 \Rightarrow n = 2$

$m - n \Rightarrow 3-2 = 1$

Q If $\dfrac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^m} = \dfrac{1}{27}$, where m and n are natural numbers, then find the value of $(m-n)$ is: (KTK 6)

Sol
$$\frac{3^{2n} \times 3^2 \times 3^n - (3^3)^n}{3^{3m} \times 2^m} = \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^m} = \frac{3^{3n}(3^2-1)}{3^{2m} \cdot 6^m}$$

$$= \frac{3^{3n} \cdot 8}{3^{2m} \cdot 6^m} = \frac{1}{27}$$

$$= \frac{3^{3n} \cdot 2^3}{3^{2m} \cdot 2^m \cdot 3^m} = \frac{1}{3^3}$$

$$3^{3n+3} \cdot 2^3 = 3^{3m} \cdot 2^m$$

$$\boxed{m=3}$$

$$3m = 3n+3$$
$$3 \times 3 = 3n+3$$
$$3n = 9-3$$
$$3n = 6$$
$$\boxed{n=2}$$

$\therefore m-n = 3-2 = 1$ Ans

QUESTION

(KTK 7)

Show that the square of $\frac{\sqrt{25-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number.

Q-10: Show that the square of $\dfrac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number.

Soln.

Method 1:

KTK 7
by Reed
from WB

$$E = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$$

or, $$E = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

or, $$E = \frac{\sqrt{52-30\sqrt{3}}}{10-\sqrt{76+10\sqrt{3}}}$$

$52 = (3\sqrt{3})^2 + 5^2$

$30\sqrt{3} = 2\cdot 15\cdot\sqrt{3}$ ✗

$= 2\cdot 5\sqrt{3}\cdot 3$ ✗

$= 2\cdot \underbrace{3\sqrt{3}\cdot 5}$ ✓

(x^2 & add $= 26\times 2 = 52$)

or, $$E = \frac{\sqrt{(3\sqrt{3})^2+5^2-2\cdot 3\sqrt{3}\cdot 5}}{10-\sqrt{(5\sqrt{3})^2+1^2+2\cdot 5\sqrt{3}\cdot 1}}$$

$76 = (5\sqrt{3})^2 + 1^2$

$10\sqrt{3} = 2\cdot \underbrace{5\sqrt{3}\cdot 1}$

(x^2 & add $= 75+1 = 76$)

or, $$E = \frac{\sqrt{(3\sqrt{3}-5)^2}}{10-\sqrt{(5\sqrt{3}+1)^2}}$$

or, $$E = \frac{|3\sqrt{3}-5|}{10-|5\sqrt{3}+1|}$$

or, $$E = \frac{3\sqrt{3}-5}{10-5\sqrt{3}-1}$$

or, $$E = \frac{3\sqrt{3}-5}{9-5\sqrt{3}}$$

or, $$E = \frac{\cancel{(3\sqrt{3}-5)}}{\sqrt{3}\,\cancel{(3\sqrt{3}-5)}} = \frac{1}{\sqrt{3}}$$

$\therefore E^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} \in \mathbb{Q}$. [PROVED]

QUESTION

(KTK 8)

If $5^{10x} = 4900, 2^{\sqrt{y}} = 25$ then the value of $\frac{\left(5^{(x-1)}\right)^5}{4^{-\sqrt{y}}}$ is

(A) $\frac{14}{5}$

(B) 5

(C) $\frac{28}{5}$

(D) 14

Ans. D

Q If $5^{10x} = 4900$, $2^{\sqrt{y}} = 25$ then the value of $\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$ is

KTK-08

Sol

$$\frac{5^{(5x-5)}}{4^{-\sqrt{y}}} = 5^{5x-5} \cdot 4^{\sqrt{y}} \Rightarrow \frac{5^{5x}\,(2^{\sqrt{y}})^2}{5^5}$$

$$\Rightarrow \frac{70 \times (25)^2}{5^5} \Rightarrow \frac{70 \times \cancel{5^4}}{\cancel{5^5}} = 14 \text{ Ans}$$

Q-11: If $5^{10x} = 4900$, $2^{\sqrt{y}} = 25$, then the value of $\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$ is:

(a) $\frac{14}{5}$ (b) 5 (c) $\frac{28}{5}$ (d) 14.

Soln

$$\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$$

$$= \frac{(5^x \times 5^{-1})^5}{(2^2)^{-\sqrt{y}}}$$

$$= \frac{5^{5x} \times 5^{-5}}{(2^{\sqrt{y}})^{-2}}$$

$$= \frac{5^{5x} \times (2^{\sqrt{y}})^2}{5^5}$$

$$= \frac{70 \times (25)^2}{5^5}$$

$$= \frac{14 \times 5 \times (5^2)^2}{5^5}$$

$$= \frac{14 \times 5 \times 5^4}{5^5}$$

$$= \frac{14 \times 5^5}{5^5}$$

$$= 14 \text{ (Ans)}$$

$\therefore$ Ans $\Rightarrow$ 14.

now,

$$5^{10x} = 4900$$

or, $5^{\frac{10x}{2}} = 4900^{\frac{1}{2}}$

or, $5^{5x} = 70$

THANK YOU